

# A Confining Non-Local Four-Fermi Interaction from Yang-Mills Theory in a Stochastic Background

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## Abstract

We derive a non-local four-fermi term with a linear potential from Yang-Mills theory in a stochastic background. The stochastic background is a class of classical configuration derived from the non-linear gauge.

## I. INTRODUCTION

The structure of the vacuum plays a key role in understanding physical phenomena. For example, QED and the asymptotic freedom phase of non-Abelian gauge theory are based on the trivial vacuum ( $A_\mu = 0$ ) and the physical vector fields, the transverse photon/gluon, are clearly exposed by using the Coulomb gauge. Confinement, on the other hand, is believed to be a non-perturbative vacuum phenomenon that is not yet clearly understood. Presently, there are two confinement mechanisms discussed extensively in the literature. These are the magnetic monopoles<sup>1,2,3,4</sup> and vortices<sup>5,6</sup>, which are related to the maximal Abelian gauge<sup>7</sup> and maximal central gauge<sup>8</sup>, respectively. Although each of the two mechanism has its own set of successes and failures, they share an important shortcoming: they are disjointed from short-distance physics. As a quark is pulled from inside a hadron, it goes through the asymptotic phase (short-distance, where transverse gluons are exchanged) to the confinement phase (large-distance, with mass gap and linearly rising interaction) eventually resulting in hadronization. Ideally then, the asymptotic phase and the confinement phase should continuously interpolate in a way where the effective degrees of freedom in each distance regimes are transparent. On the lattice, this problem is dealt with using a decimation procedure.<sup>9</sup> For a number of years now, this author has been claiming that this is achieved in the continuum by using the non-linear gauge.

The non-linear gauge<sup>10</sup>, which is given by

$$(\partial \cdot D)^{ab}(\partial \cdot A^b) = (D \cdot \partial)^{ab}(\partial \cdot A^b) = (\partial^2 \delta^{ab} - g\epsilon^{abc} A^c \cdot \partial)(\partial \cdot A^b) = 0, \quad (1)$$

was proposed by the author eleven years ago because of the observation that field configurations that satisfy  $\partial \cdot A^a = f^a(x) \neq 0$  and equation (1) cannot be gauge-transformed to the Coulomb gauge. This follows from the fact that the zero mode of  $(\partial \cdot D)^{ab}$ , which is  $\partial \cdot A^a$ , is also the source in the equation that solves for the gauge parameter. And conversely, field configurations on the Coulomb surface cannot be gauge transformed to the non-linear regime of equation (1), i.e., there is no  $\Lambda^a$  such that  $A'_\mu = A_\mu + D_\mu \Lambda$ ,  $\partial \cdot A' \neq 0$ , and  $(\partial \cdot D(A'))(\partial \cdot A') = 0$ . These mean that the two regimes of equation (1), which are (i) the linear regime with  $\partial \cdot A^a = 0$ , and (ii) the non-linear regime, which effectively is the Gribov horizon of the surface  $\partial \cdot A^a = f^a(x)$ , do not mix. Thus, the Coulomb gauge is an incomplete gauge-fixing for non-Abelian theory, although there are claims to the contrary.<sup>11</sup>

Using the running of the coupling constant, the author argued that the non-linear gauge interpolates between short-distance (perturbative) physics and large-distance (non-perturbative) physics. In short-distance regime, where the Coulomb gauge is in effect, the transverse gluons are the physical degrees of freedom. However, in the large-distance regime, the vector field  $t_\mu^a$  and the scalar field  $f^a$ , which satisfy two sets of constraints, are the effective degrees of freedom.<sup>12</sup> The full quantum dynamics of the scalars lead to a Parisi-Sourlas mechanism<sup>13</sup> with an  $O(1,3)$  symmetry<sup>14</sup>, which unfortunately has a wrong sign for the kinetic term of the scalars. Thus, although we have shown equivalence to a 2D  $O(1,3)$  non-linear sigma model, the proof of confinement remains formal because aside from the wrong sign of the scalar kinetic term, the mechanism is not identified and quarks and "gluons" are not involved.

To identify the mechanism, the author proposes that we look for (a) classical solution(s) of the pure  $f^a$  dynamics which is (are) vacuum configuration(s). We then expand on this (these) configuration(s) and see the physics of quarks, "gluons" and scalar fluctuations around the background. This program was started by the author about five years ago and the following results had been derived. First, that all spherically symmetric functions  $\tilde{f}^a(x)$ , with  $x = (x_\mu x_\mu)^{1/2}$ , in  $R^4$  are classical solutions with zero field strength.<sup>15</sup> This shows that all the  $\tilde{f}^a(x)$  are vacuum configurations with zero action, which means the configuration space has a very broad minimum.

Second, since we do not have just one classical configuration but a whole class of solutions, the author proposed to treat them as a stochastic background with a white-noise distribution. This led to an area law behaviour for the Wilson loop.<sup>15</sup> Thus, we have identified a possible mechanism for confinement.

Third, when we consider the classical dynamics of the "gluons" in the large distance regime, we find the gluons acquiring a mass.<sup>16</sup> This verifies the existence of a mass gap. Furthermore, the "gluons" also lose their self-interaction. This result was also arrived at by Kondo by a suitable redefinition of fields.<sup>17</sup>

In a forthcoming paper<sup>18</sup>, which will provide the details of the results presented here and discuss quantum field theory in a stochastic background, the author will show that the scalars  $\phi^a(x)$ , which are fluctuations off the classical stochastic background  $\tilde{f}^a(x)$ ,

$$f^a = \tilde{f}^a(x) + \phi^a(x) \tag{2}$$

are non-propagating, i.e., they do not have a kinetic term. As a matter of fact, we will show that  $\langle S_{YM}(A = A(\tilde{f} + \phi, t_\mu^a)) \rangle_{\tilde{f}}$  is independent of the scalars  $\phi^a$  and only dependent on  $t_\mu^a$ . This resolves the problem of scalars with a wrong sign for the kinetic term encountered in the derivation of the Parisi-Sourlas mechanism, where the full quantum theory of the scalars was shown to be equivalent to a 2D  $O(1,3)$  non-linear  $\sigma$  model.

In this paper, we will present the quantum dynamics of quarks and "gluons" in a stochastic background, resulting mainly in the derivation of a non-local four-fermi interaction with a linear potential.

## II. QUARKS AND "GLUONS" IN THE NON-LINEAR REGIME

In this section, we will provide the background needed to derive the main result which will be discussed in section 3.

Consider  $SU(2)$  theory with action

$$S = S_{YM} + S_{fermion} = \int d^4x \left[ \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \bar{\Psi} i \gamma_\mu D_\mu \Psi \right] = \int \mathcal{L} d^4x \quad (3)$$

The gauge-fixing given by equation(1) leads to the resolution of the potential in the non-linear regime as given by

$$A_\mu^a(x) = \frac{1}{(1 + \vec{f} \cdot \vec{f})} (\delta^{ab} + \epsilon^{abc} f^c + f^a f^b) \left( \frac{1}{g} \partial_\mu f^b + t_\mu^b \right) \quad (4)$$

The scalars  $f^a$  and the "gluon"  $t_\mu^a$  satisfy the following constraints, which guarantee the same number of degrees of freedom,

$$\partial \cdot t^a - \frac{1}{g\ell^2} f^a = 0, \quad (5)$$

$$\begin{aligned} \rho^a &= \frac{1}{(1 + \vec{f} \cdot \vec{f})^2} [\epsilon^{abc} + \epsilon^{abd} f^d f^c - \epsilon^{acd} f^d f^b + f^a f^d \epsilon^{dbc} \\ &\quad - f^a (1 + \vec{f} \cdot \vec{f}) \delta^{bc} - f^c (1 + \vec{f} \cdot \vec{f}) \delta^{ab}] \partial_\mu f^b t_\mu^c = 0. \end{aligned} \quad (6)$$

Substituting equation (4) in the field strength, we find

$$F_{\mu\nu}^a = \frac{1}{g} Z_{\mu\nu}^a(f) + L_{\mu\nu}^a(f; t) + g Q_{\mu\nu}^a(f; t), \quad (7)$$

$$Z_{\mu\nu}^a = X^{abc} \partial_\mu f^b \partial_\nu f^c, \quad (8)$$

$$L_{\mu\nu}^a(f, t) = R^{ab} (\partial_\mu t_\nu^b - \partial_\nu t_\mu^b) + Y^{abc} (\partial_\mu f^b t_\nu^c - \partial_\nu f^b t_\mu^c), \quad (9)$$

$$Q_{\mu\nu}^a(f; t) = T^{abc} t_\mu^b t_\nu^c, \quad (10)$$

$$X^{abc} = \frac{1}{(1 + \vec{f} \cdot \vec{f})^2} [-(1 + 2\vec{f} \cdot \vec{f})\epsilon^{abc} + 2\delta^{ab}f^c - 2\delta^{ac}f^b + 3\epsilon^{abd}f^d f^c - 3\epsilon^{acd}f^d f^b + \epsilon^{bcd}f^a f^d], \quad (11)$$

$$R^{ab} = \frac{1}{(1 + \vec{f} \cdot \vec{f})} (\delta^{ab} + \epsilon^{abc}f^c + f^a f^b), \quad (12)$$

$$Y^{abc} = \frac{1}{(1 + \vec{f} \cdot \vec{f})^2} [-(\vec{f} \cdot \vec{f})\epsilon^{abc} + (1 + \vec{f} \cdot \vec{f})f^a \delta^{bc} - (1 - \vec{f} \cdot \vec{f})\delta^{ac}f^b + 3\epsilon^{cad}f^d f^b - 2f^a f^b f^c + \epsilon^{abd}f^d f^c + f^a \epsilon^{bcd}f^d], \quad (13)$$

$$T^{abc} = \frac{1}{(1 + \vec{f} \cdot \vec{f})} [\epsilon^{abc} + (1 + \vec{f} \cdot \vec{f})f^b \delta^{ac} - (1 + \vec{f} \cdot \vec{f})\delta^{ab}f^c + \epsilon^{abd}f^d f^c + f^a \epsilon^{bcd}f^d + \epsilon^{adc}f^d f^b]. \quad (14)$$

The pure  $f^a$  dynamics given by the action

$$S_1(f) = \frac{1}{4g^2} \int d^4x Z_{\mu\nu}^a Z_{\mu\nu}^a, \quad (15)$$

hints of non-perturbative physics because (i) of the  $1/g^2$  factor that goes with  $S_1$ , (ii) the action is infinitely non-linear, and (iii)  $S_1 \sim (\partial f)^4$ . For these reasons, we look for a classical solution to

$$\frac{\delta S_1}{\delta f^a} = \frac{\delta Z_{\mu\nu}^b}{\delta f^a} Z_{\mu\nu}^b = 0. \quad (16)$$

From equation (8) and (11), we find that all spherically  $\tilde{f}^a(x)$ , with  $x = (x_\mu x_\mu)^{1/2}$  are classical solutions with zero field strength  $Z_{\mu\nu}^a$ . This follows from  $\partial_\mu \tilde{f}^a = \frac{x_\mu}{x} \frac{d\tilde{f}^a}{dx}$  and the anti-symmetry of  $X^{abc}$  with respect to b and c. Thus, we are dealing with a whole class of classical solutions with zero field strength. This shows that the action has a very broad bottom (note  $S_1$  is positive semi-definite).

Since we do not have just one solution but a whole class of solutions, it was proposed in reference (7) to treat  $\tilde{f}^a(x)$  as a stochastic background with a white-noise distribution

$$P[\tilde{f}] = \mathcal{N} \exp.(-\frac{1}{\ell} \int_0^\infty \tilde{f}^a(s) \tilde{f}^a(s) ds). \quad (17)$$

We identify  $\ell$  as the length scale where non-perturbative physics becomes important. From the running of the coupling constant, we deduce  $\ell = \Lambda_{QCD}^{-1}$ . Substituting the background decomposition given by equation (2) in  $S_{YM}$ , we will get a rather involved expression for the action of the "gluon"  $t_\mu^a$  and the scalars  $\phi^a$  in the presence of the background  $\tilde{f}^a$ . However, as we show in reference (18), all the stochastic averages drop out except the following simple

result

$$\langle S_{YM} \rangle_{\bar{f}} = \frac{1}{4} \int d^4x \left\{ \frac{1}{3} (\partial_\mu t_\nu^a - \partial_\nu t_\mu^a)^2 + \frac{3}{2} \left( \frac{n}{\ell} \right)^2 t_\mu^a t_\mu^a \right\}. \quad (18)$$

This result shows that the scalars  $\phi^a$  are non-propagating and the "gluons"  $t_\mu^a$  acquired a mass and lost their self-interactions.

The stochastic average of the fermionic term, on the other hand, yields

$$\langle S_{fermion} \rangle_{\bar{f}} = \int d^4x \bar{\psi} i \gamma_\mu [\partial_\mu - g T^a \left( \frac{1}{3} t_\mu^a + \frac{1}{3g} \partial_\mu \phi^a - \frac{2}{3} g \left( \frac{n}{\ell} \right) \frac{x_\mu}{x} \phi^a \right)] \psi(x). \quad (19)$$

We can remove the scalars from equation (19) by using the stochastic average of the constraints given by equations (5) and (6), which are

$$\partial \cdot t^a = \frac{1}{g \ell^2} \phi^a, \quad (20)$$

$$x \cdot t^a = 0 \quad (21)$$

Substituting equation (20) in equation (19), we find that stochastically averaged effective dynamics yield the following for fermi interaction

$$\begin{aligned} S_{FF} &= g^2 \int d^4x d^4y (\bar{\psi} i \gamma_\mu T^a \psi)_x \left( \frac{1}{3} \delta_{\mu\alpha} + \frac{1}{3} \ell^2 \overrightarrow{\partial}_\mu \overrightarrow{\partial}_\alpha - \frac{2}{3} g^2 \left( \frac{n}{\ell} \right)^2 \frac{x_\mu}{x} \overrightarrow{\partial}_\alpha \right)_x \\ &\langle 0 | T(t_\alpha^a(x) t_\beta^{a'}(y)) | 0 \rangle \left( \frac{1}{3} \delta_{\nu\beta} + \frac{1}{3} \ell^2 \overleftarrow{\partial}_\nu \overleftarrow{\partial}_\beta - \frac{2}{3} g^2 \left( \frac{n}{\ell} \right)^2 \frac{y_\nu}{y} \overleftarrow{\partial}_\beta \right)_y (\bar{\psi} i \gamma_\nu T^{a'} \psi)_y \end{aligned} \quad (22)$$

The Greens function  $G_{\alpha\beta}(x-y)$ , which is the propagator of the "gluons" as given by

$$\begin{aligned} \delta^{aa'} G_{\alpha\beta}(x-y) &= \langle 0 | T | t_\alpha^a(x) t_\beta^{a'}(y) | 0 \rangle \\ &= \delta^{aa'} \delta_{\alpha\beta} \tilde{G}(x-y) - \frac{1}{m^2} \delta^{aa'} \partial_\alpha^x \partial_\beta^x \tilde{G}(x-y), \end{aligned} \quad (23)$$

where  $m = \frac{n}{\ell}$  and  $\tilde{G}(x-y)$  satisfies<sup>20</sup>

$$(-\square^2 + m^2) \tilde{G}(x-y) = \delta^4(x-y). \quad (24)$$

The solution to equation (24) for large  $|x-y|$  is<sup>19</sup>

$$\tilde{G}(x-y) \sim \frac{1}{|x-y|^{\frac{3}{2}}} e^{-m|x-y|}. \quad (25)$$

Substituting equations (25), (23) in equation (22), we find that the effective four-fermi term from the "gluon" exchange does not confine because the exponential decay behaviour dominates. The confining non-local four-fermi term must come from elsewhere, which must involve the vacuum configuration at the outset.

### III. DERIVATION OF THE CONFINING FOUR-FERMI INTERACTION

The path-integral in the non-linear regime of the non-linear gauge is given by<sup>14</sup>

$$W = \int (dt_\mu^a)(df^a)(d\psi)(d\bar{\psi})\delta(\partial \cdot t^a - \frac{1}{g\ell^2}f^a)\delta(\rho^a) \\ \times \det^4(1 + \vec{f} \cdot \vec{f}) \det \theta \exp.\{-(S_{YM} + S_{fermion})\} \quad (26)$$

where  $\rho^a$  is given by equation (6) and  $\Theta$  is the dimension 4 "Fadeev-Popov" operator<sup>15</sup> given by

$$\theta^{ad} = (D \cdot \partial)^{ab}(\partial \cdot D)^{bd} - \frac{1}{g\ell^2}\epsilon^{abc}(\partial_\mu f^b \cdot D_\mu^{cd}). \quad (27)$$

Since we will do a background decomposition as given by equation (2) to find the vacuum to vacuum functional in the presence of  $\tilde{f}^a(x)$  and then do a stochastic averaging, we will change the delta functionals by

$$\delta(\partial \cdot t^a - \frac{1}{g\ell^2}f^a) = \det(\frac{1}{(1 + \vec{f} \cdot \vec{f})^j})\delta(\frac{1}{(1 + \vec{f} \cdot \vec{f})^j}(\partial \cdot t^a - \frac{1}{g\ell^2}f^a)) \\ \delta(\rho^a) = \det(\frac{1}{(1 + \vec{f} \cdot \vec{f})^k})\delta(\frac{1}{(1 + \vec{f} \cdot \vec{f})^k}\rho^a) \quad (28)$$

where the powers  $j$  and  $k$  can be freely chosen so that stochastic averages related to the gauge-fixing and the "Fadeev-Popov" determinant vanish.

To see how this happens, let us write the path-integral as

$$W = \int (dt_\mu^a)(df^a)(d\psi)(d\bar{\psi})(du^a)(d\bar{u}^a)\exp.\{-S'\}, \quad (29)$$

where

$$S' = \int d^4x \mathcal{L}'(x),$$

$$\mathcal{L}'(x) = \mathcal{L}_{YM} + \mathcal{L}_{fermion} + \mathcal{L}_{gf} + \mathcal{L}_{FP} \quad (30)$$

$$\mathcal{L}_{gf} = (\frac{1}{\alpha})\frac{1}{(1 + \vec{f} \cdot \vec{f})^{2j}}(\partial \cdot t^a - \frac{1}{g\ell^2}f^a)^2 + (\frac{1}{\beta})\frac{1}{(1 + \vec{f} \cdot \vec{f})^{2k}}\rho^a\rho^a \quad (31)$$

$$\mathcal{L}_{FP} = \bar{u}^a \frac{1}{[(1 + \vec{f} \cdot \vec{f})^{j+k+4}]} \theta^{ab} u^b. \quad (32)$$

The ghosts  $\bar{u}^a$ ,  $u^a$  are introduced to express the determinants. Next, we introduce the background decomposition given by equation (2) and equation (29) will yield

$$W[\tilde{f}^a] = \int (dt_\mu^a)(d\psi)(d\bar{\psi})(d\phi^a)(d\bar{u}^a)(du^a)\exp.\{-S'(t, \phi, \psi, \bar{\psi}, u^a, \bar{u}^a; \tilde{f})\} \quad (33)$$

Next we do the stochastic averaging given by

$$\langle W[\tilde{f}^a] \rangle_{\tilde{f}^a} = \int (dt_\mu^a)(d\psi)(d\bar{\psi})(d\phi^a)(du^a)(d\bar{u}^a) \langle \exp.\{-S'\} \rangle_{\tilde{f}} \quad (34)$$

We evaluate the stochastic average by expanding  $e^{-s'}$ . The following averages are needed:

$$\langle S' \rangle = \langle S_{YM} + S_{fermion} + S_{gf} + S_{FP} \rangle_{\tilde{f}}, \quad (35)$$

$$\langle S'^2 \rangle = \langle S' \rangle^2 + \int d^4x d^4y \langle \underbrace{\mathcal{L}'(x)\mathcal{L}'(y)}_{\tilde{f}} \rangle_{\tilde{f}}, \quad (36)$$

$$\langle S'^3 \rangle = \langle S' \rangle^3 + 3 \left( \int d^4x d^4y \langle \underbrace{\mathcal{L}'(x)\mathcal{L}'(y)}_{\tilde{f}} \rangle_{\tilde{f}} \right) \langle S' \rangle, \quad (37)$$

$$\begin{aligned} \langle S'^4 \rangle &= \langle S' \rangle^4 + 3 \left( \int d^4x d^4y \langle \underbrace{\mathcal{L}'(x)\mathcal{L}'(y)}_{\tilde{f}} \rangle_{\tilde{f}} \right) \langle S' \rangle^2, \\ &+ \int d^4x d^4y d^4z d^4r \langle \underbrace{\mathcal{L}'(x)\mathcal{L}'(y)\mathcal{L}'(z)\mathcal{L}'(r)}_{\tilde{f}} \rangle_{\tilde{f}}. \end{aligned} \quad (38)$$

In equation (35), we make use of equations (18) and (19). In equations (36,37) and (38), the symbols  $\underbrace{\mathcal{L}'(x)\mathcal{L}'(y)}_{\tilde{f}}$ , etc., represent the correlated points, which arise because the derivative of the white-noise  $\tilde{f}^a(x)$  is "smoothed out" via

$$\frac{d\tilde{f}^a}{dx} = \frac{\tilde{f}^a(x + \frac{\ell}{n}) - \tilde{f}^a(x)}{\frac{\ell}{n}} \quad (39)$$

Note that all odd correlations, such as  $\langle \underbrace{\mathcal{L}'(x)\mathcal{L}'(y)\mathcal{L}'(z)}_{\tilde{f}} \rangle_{\tilde{f}}$  vanish because of the white-noise character of  $\tilde{f}^a(x)$ . The stochastic averages involve the following integral

$$\lim_{\sigma \rightarrow 0} \pi^{-3/2} \sigma^{+3/2} \int_0^\infty \frac{r^{2m}}{(1+r^2)^n} e^{-\sigma r^2} dr = \begin{cases} 0, & \text{for } m \leq n \\ \text{finite}, & \text{for } m = n + 1 \\ \text{diverges}, & \text{for } m \geq 0, n + 2. \end{cases} \quad (40)$$

Using equation (40) and because of the  $(1 + \tilde{f} \cdot \tilde{f})^2$  factors in the denominators of  $S_{gf}$  and  $S_{FP}$ , we find that when we do a background decomposition given by equation (2), the stochastic averages of each of the expansion terms vanish. Thus, we find

$$\langle S_{gf} \rangle_{\tilde{f}} = \langle S_{FP} \rangle_{\tilde{f}} = 0, \quad (41)$$

giving  $\langle S' \rangle = \langle S \rangle$ , which is given by equations (18) and (19). Furthermore, the correlated terms in equations (36), (37) and (38) that involve  $\mathcal{L}_{gf}$  and  $\mathcal{L}_{FP}$  also vanish for the same reason.

Taking everything so far into account, we find

$$\begin{aligned}
\langle e^{-S'} \rangle_{\bar{f}} &= 1 - \langle S \rangle + \frac{1}{2} [\langle S \rangle^2 \\
&+ \int d^4x d^4y \langle \underbrace{\mathcal{L}(x) \mathcal{L}(y)}_{\bar{f}} \rangle - \frac{1}{3!} [\langle S \rangle^3 \\
&+ 3 [(\int d^4x d^4y) (\langle \underbrace{\mathcal{L}(x) \mathcal{L}(y)}_{\bar{f}} \rangle \langle S \rangle)] \\
&+ \frac{1}{4!} [\langle S \rangle^4 + 6 (\int d^4x d^4y \langle \underbrace{\mathcal{L}(x) \mathcal{L}(y)}_{\bar{f}} \rangle \langle S \rangle^2 \\
&+ \int d^4x d^4y d^4z d^4r \langle \underbrace{\mathcal{L}(x) \mathcal{L}(y) \mathcal{L}(z) \mathcal{L}(r)}_{\bar{f}} \rangle] \\
&+ \dots
\end{aligned} \tag{42}$$

Summing the series gives

$$\langle e^{-S'} \rangle_{\bar{f}} = e^{-S_{eff}}, \tag{43}$$

where

$$\begin{aligned}
S_{eff} &= \langle S \rangle - \frac{1}{2} \int d^4x d^4y \langle \underbrace{\mathcal{L}(x) \mathcal{L}(y)}_{\bar{f}} \rangle \\
&- \frac{1}{4!} \int d^4x d^4y d^4z d^4r \langle \underbrace{\mathcal{L}(x) \mathcal{L}(y) \mathcal{L}(z) \mathcal{L}(r)}_{\bar{f}} \rangle \\
&+ \dots
\end{aligned} \tag{44}$$

Since equation (44) is ghost independent, we simply drop the ghost path-integral and lump it with the normalization factors needed to make sense of the path-integral.

At this point, where is the non-local four-fermi (NLFF) interaction in equation (44)? It is found in

$$NLFF = \frac{g^2}{2} \int d^4x d^4y (\bar{\psi} \gamma_\mu T^a \psi)_x \langle A_\mu^a(x) A_\nu^b(y) \rangle_{\bar{f}} (\bar{\psi} \gamma_\nu T^b \psi)_y. \tag{45}$$

We will evaluate the stochastic average by making use of

$$\partial_\mu A_\mu^a(x) = \frac{1}{g\ell^2} f^a(x) = \frac{1}{g\ell^2} \tilde{f}^a(x) + \frac{1}{g\ell^2} \phi^a(x). \tag{46}$$

From equation (46), we must have

$$\partial_\mu^x \partial_\nu^y \langle A_\mu^a(x) A_\nu^b(y) \rangle_{\bar{f}} = \frac{1}{g^2 \ell^3} \delta(x - y) + \dots, \tag{47}$$

where  $x = (x_\mu x_\mu)^{1/2}$  and  $y = (y_\mu y_\mu)^{1/2}$ . Equation (47) implies that

$$\langle A_\mu^a(x) A_\nu^b(y) \rangle = \left( \frac{1}{g^2 \ell^3} \right) \frac{x_\mu y_\nu}{x y} \delta^{ab} |x - y| + \dots \tag{48}$$

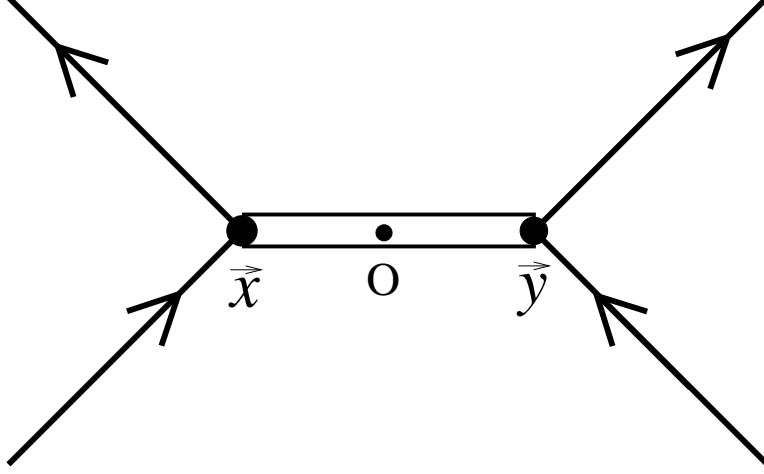


FIG. 1: The four-fermi term with interaction constrained with  $\vec{x}, \vec{y}$  collinear hinting of a flux tube geometry

The equivalence follows from the fact that for a spherically symmetric function,  $\partial_\mu^x = \frac{x_\mu}{x} \frac{d}{dx}$  and  $\frac{d^2}{dx^2}|x - y| = \delta(x - y)$ . Substituting equation (47) in NLFF, we find

$$NLFF = \frac{1}{2} \left( \frac{1}{\ell^3} \right) \int d^4x d^4y (\bar{\psi} \eta_\mu \gamma_\mu T^a \psi)_x |x - y| (\bar{\psi} \gamma_\nu \eta_\nu T^a \psi)_y + \dots \quad (49)$$

where

$$\eta_\mu = (\sin\theta_1 \sin\theta_2 \sin\phi, \sin\theta_1 \sin\theta_2 \cos\phi, \sin\theta_1 \cos\theta_2, \cos\theta_1), \quad (50)$$

i.e.,  $\eta_\mu$  represents the unit vectors in 4D spherical coordinates. If the fermion field is spherically symmetric, the angular integration does not vanish only when  $\eta_\mu(\vec{x}) = \pm \eta_\mu(\vec{y})$ , i.e., the 4D vectors are collinear. Using

$$\int d\Omega_4 \eta_\mu \eta_\nu = \frac{\pi^2}{2} \delta_{\mu\nu} \quad (51)$$

and  $\int d\Omega_4 = \int_0^{2\pi} \int_0^\pi \int_0^\pi \sin^2 \theta_1 \sin \theta_2 d\theta_1 d\theta_2 d\phi = 2\pi^2$ , we find that we can write the equation (49) as

$$NLFF = \frac{1}{8} \frac{1}{\ell^3} \int d^4x d^4y (\bar{\psi} \gamma_\mu \psi)_x |\vec{x} - \vec{y}| (\bar{\psi} \gamma_\mu \psi)_y \quad (52)$$

with  $\vec{x}$  and  $\vec{y}$  collinear. This hints of flux tube geometry as shown in figure 1:

This result may be a bit surprising because we started with an expansion about a classical background  $\tilde{f}^a(x)$ , which is spherically symmetric. But considering that the  $r$  potential is no less or no more spherically symmetric than the  $\frac{1}{r}$  term, this result is not unreasonable at all.

When we determined the correlation  $\langle A_\mu^a(x) A_\nu^b(y) \rangle_{\tilde{f}}$  to arrive at the linear term (note there are extra terms), we made use of components of the vector field involving  $\phi^a, t_\mu^a$  in a background  $\tilde{f}^a(x)$ . All these components are important for the following reasons: (1) As was shown in Section II, just considering  $t_\mu^a$  alone after stochastic averaging, does not lead to a confining term. (2) As for the  $\phi^a$  alone, it has no kinetic term and if we integrate it out yields a constraint on the fermion fields, which under the assumption of spherical symmetry says the fermion field must vanish beyond  $\ell$  (see reference (18)). (3) Considering  $\tilde{A}_\mu^a(\tilde{f})$  alone will not do either because  $\partial_\mu \tilde{A}_\mu^a(\tilde{f}) = 0$ , if we neglect  $\phi^a$  and  $t_\mu^a$ , and we needed the  $\tilde{f}_\mu^a(x)$  at the right-hand side of equation (46) to derive equation (48). Thus, a certain combination of  $\phi^a, t_\mu^a$  in  $\tilde{f}^a(x)$  is key in deriving the confining non-local four-fermi term. Determining which components these are is difficult from a direct evaluation because there are many terms that contribute to it. The linear term appears but an unusual "renormalization", to get rid of a  $\frac{1}{\sigma}$  term with  $\sigma \rightarrow 0$ , must be carried out. Thus, the derivation made use of equation (46) where, the divergence does not show up.

#### IV. CONCLUSION

In this paper, we showed confinement even with dynamical quarks. The confinement mechanism is the stochastic treatment of the scalar classical configurations  $\tilde{f}^a(x)$ , which arise in the non-linear regime of the non-linear gauge.

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<sup>1</sup> Y. Nambu, Phys. Rev. **D10**, 4262 (1974).

<sup>2</sup> G. Parisi, Phys. Rev. **D11**, 970 (1975).

<sup>3</sup> G. Mandelstam, Physics Reports **23**, 245 (1976).

<sup>4</sup> G. 't Hooft, *Proceedings of the EPS International Conference* (Palermo, Italy, 1976).

- <sup>5</sup> G. 't Hooft, Nuclear Physics **B138**, 1 (1978).
- <sup>6</sup> G. Mack and V. Petkova, Ann. Phys (**NY**)**123**, 442 (1979).
- <sup>7</sup> G. 't Hooft, Nucl. Phys **B190**, 455 (1981).
- <sup>8</sup> L. del Debbio, M. Faber, J. Greensite, and S. Olejnik, Phys. Rev. **D55**, 2298 (1997).
- <sup>9</sup> E. Tomboulis, **hep-lat/0311022** (17 November 2003).
- <sup>10</sup> J. A. Magpantay, Progress of Theoretical Physics **91**, 573 (1994).
- <sup>11</sup> G. D. Antonio and D. Zwanziger, Com. Math. Physics **138**, 291 (1991).
- <sup>12</sup> J. A. Magpantay, in *Mathematical Methods of Quantum Physics: Essays in honor of Hiroshi Ezawa*, Ed. C.C. Bernido, et al (Gordon and Breach Science, 1999).
- <sup>13</sup> G. Parisi and N. Surlas, Phys. Rev. Lett **43**, 744 (1979).
- <sup>14</sup> J. A. Magpantay, International Journal of Modern Physics **A15**, 1613 (2000).
- <sup>15</sup> J. A. Magpantay, Modern Physics Letters **A14**, 447 (1999).
- <sup>16</sup> J. A. Magpantay, hep-th/0203178 **version 2** (11 July 2003).
- <sup>17</sup> K. Kondo, hep-th/0311033 **version 2** (23 Jan. 2004).
- <sup>18</sup> J. A. Magpantay, hep-th/0412121 (????).
- <sup>19</sup> I. Gradshteyn and M. Ryzhik, *Table of Integrals, Series and Products*, 419 and 687 (Academic Press, USA, 1965), 2nd ed.
- <sup>20</sup> In reference<sup>16</sup>, the negative sign from integration by parts was omitted resulting in the wrong sign ( $+\square^2$ ) instead of  $-\square^2$  in field equation. Furthermore, since the scalar fluctuations  $\phi^a$  were neglected, the gluons satisfied the constraint  $\partial_\mu t_\mu^a = 0$ , instead of equation 20, thus resulting in an inconsistent massive but transverse gluons in reference<sup>16</sup>. This is now corrected in this paper.